AS - 2824

Model Answer

B.Sc. (Hon's) (Third Semester) Examination, 2013

Mathematics Paper - Second (Geometry)

1. (i) Director Circle : Locus of the point of intersection of two perpendicular tangents of a conic is called Director Circle of a conic.

Equation of tangent at point 'α' of conic $\frac{l}{r} = 1 + e\cos(\theta - \alpha)$ is $\frac{l}{r} = e\cos(\theta - \alpha) + \cos(\theta - \alpha)$

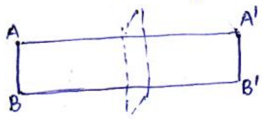
(ii) Angle between two planes : If $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ be the equations of two planes and θ be the angle between the planes then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}$$

(iii) If α, β, γ be the angles which a line makes with positive direction of X, Y, Z axis then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\left(\because l^2+m^2+n^2=1 \text{ where } l=\cos\alpha,\ m=\cos\beta,\ n=\cos\gamma\right)$

$$\Rightarrow 1-\sin^2\alpha + 1-\sin^2\beta + 1-\sin^2\gamma = 1$$
$$3-(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$$
$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

(iv) Image of a Line in a plane : Image of a given line in given plane is defined as line joining the image points of any two points on the line in that plane.



(V) (a) Equation of plane in intercept on axis form is
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

(b) Equation of line in non-symmetrical form :

Intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ represent a line.

(VI) Radical Plane : Radical plane of two spheres is defined as locus of points whose powers with respect to two spheres are equal.

Radical plane of two spheres $S_1 = 0$, $S_2 = 0$ is given by

(vii) Equation of the cone whose vertex is at origin is a homogeneous equation of second degree.

Making homogeneous to $ax^2+by^2+cz^2=1$ with the help of $lx+my+nz=p$ we can get the eqn of cone.

$$lx+my+nz=p \Rightarrow \frac{lx+my+nz}{p}=1 \Rightarrow \left(\frac{lx+my+nz}{p}\right)^2=1$$

Put $1=\left(\frac{lx+my+nz}{p}\right)^2$ in eqn $ax^2+by^2+cz^2=1$

we have $$ax^2+by^2+cz^2=\frac{(lx+my+nz)^2}{p^2}$$

$$\Rightarrow p^2(ax^2+by^2+cz^2)=(lx+my+nz)^2$$

$$\Rightarrow ap^2x^2+bp^2y^2+cp^2z^2=l^2x^2+m^2y^2+n^2z^2+2lmxy+2mnyz+2lnxz$$

$$\Rightarrow (ap^2-l^2)x^2+(bp^2-m^2)y^2+(cp^2-n^2)z^2-2lmxy-2mnyz-2lnxz=0$$

(viii) Equation of hyperboloid on one sheet is $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$.

2. Equation of conic (parabola) is $\frac{l}{r}=1+\cos\theta$ ——— ①

equation of normal at point 'θ_1' is

$$\frac{l\sin\theta_1}{r(1+\cos\theta_1)}=\sin\theta+\sin(\theta-\theta_1)$$

If Normal passes through the point (ρ,ϕ) then

$$\frac{l\sin\theta_1}{\rho(1+\cos\theta_1)}=\sin\phi+\sin(\phi-\theta_1)$$

$$\frac{2\cdot l\sin\frac{\theta_1}{2}\cos\frac{\theta_1}{2}}{\rho(\cancel{1}+2\cos^2\frac{\theta_1}{2}\cancel{-1})}=\sin\phi+\sin\phi\cos\theta_1-\cos\phi\sin\theta_1$$

$$\left(\frac{l}{\rho}\right)\cdot\tan\frac{\theta_1}{2}=\sin\phi(1+\cos\theta_1)-\cos\phi\sin\theta_1$$

$$\frac{l}{\rho}\cdot\tan\frac{\theta_1}{2}=\sin\phi\cdot 2\cos^2\frac{\theta_1}{2}-\cos\phi\cdot 2\sin\frac{\theta_1}{2}\cos\frac{\theta_1}{2}$$

$$\frac{l}{\rho}\cdot\tan\frac{\theta_1}{2}\cdot\sec^2\frac{\theta_1}{2}=2\sin\phi-2\cos\phi\cdot\tan\frac{\theta_1}{2}$$

$$\frac{l}{\rho}\tan\frac{\theta_1}{2}\left(1+\tan^2\frac{\theta_1}{2}\right)=2\sin\phi-2\cos\phi\cdot\tan\frac{\theta_1}{2}$$

$$\left(\frac{l}{\rho}\right)\tan^3\frac{\theta_1}{2}+\left(\frac{l}{\rho}+2\cos\phi\right)\tan\frac{\theta_1}{2}-2\sin\phi=0$$

This is cubic equation in $\tan\frac{\theta_1}{2}$, gives 3 values of $\frac{\theta_1}{2}$ say $\tan\frac{\alpha}{2}$, $\tan\frac{\beta}{2}$, $\tan\frac{\gamma}{2}$.

Let $S_1\equiv\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=0$

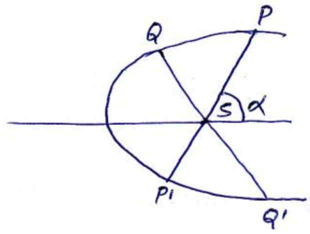
$S_2\equiv\tan\frac{\alpha}{2}\tan\frac{\beta}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\alpha}{2}\tan\frac{\gamma}{2}=\frac{l/\rho+2\cos\phi}{l/\rho}=1+\frac{2\rho}{l}\cos\phi$

$S_3\equiv\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}=\frac{2\sin\phi}{l/\rho}=\frac{2\rho}{l}\sin\phi$

We know that $\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2}\right)=\frac{S_1-S_3}{1-S_2}=\frac{0-\frac{2\rho}{l}\sin\phi}{\cancel{1}-\cancel{1}-\frac{2\rho}{l}\cos\phi}=\tan\phi$

$\frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2}=\phi \Rightarrow \boxed{\alpha+\beta+\gamma=2\phi}$

3. (a) PSP' and QSQ' be the two perpendicular focal chords of conic $\frac{l}{r} = 1 + e\cos\theta$ —— ①



Polar coordinates of $P(SP, \alpha)$, $P'(SP', \pi+\alpha)$

$Q(SQ, \frac{\pi}{2}+\alpha)$, $Q'(SQ', \frac{3\pi}{2}+\alpha)$

Now points P, P', Q, Q' lie on conic ①

$$\Rightarrow \frac{l}{SP} = 1 + e\cos\alpha \quad \Rightarrow SP = \frac{l}{1+e\cos\alpha}$$

$$\frac{l}{SP'} = 1 + e\cos(\pi+\alpha) \quad \Rightarrow SP' = \frac{l}{1-e\cos\alpha}$$

$$\frac{l}{SQ} = 1 + e\cos(\frac{\pi}{2}+\alpha) \quad \Rightarrow SQ = \frac{l}{1-e\sin\alpha}$$

$$\frac{l}{SQ'} = 1 + e\cos(\frac{3\pi}{2}+\alpha) \quad \Rightarrow SQ' = \frac{l}{1+e\cos(3\pi/2+\alpha)} = \frac{l}{1+e\sin\alpha}$$

Sum of the reciprocals of perpendicular focal chords PP' and QQ' will be

$$= \frac{1}{PP'} + \frac{1}{QQ'} = \frac{1}{SP+SP'} + \frac{1}{SQ+SQ'}$$

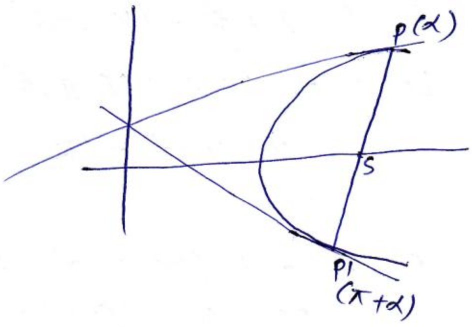
$$= \frac{1}{\frac{l}{1+e\cos\alpha} + \frac{l}{1-e\cos\alpha}} + \frac{1}{\frac{l}{1-e\sin\alpha} + \frac{l}{1+e\sin\alpha}}$$

$$= \frac{1-e^2\cos^2\alpha}{l(1-\cancel{e\cos\alpha}+1+\cancel{e\cos\alpha})} + \frac{1-e^2\sin^2\alpha}{l(1+\cancel{e\sin\alpha}+1-\cancel{e\sin\alpha})}$$

$$= \frac{2-e^2(\cos^2\alpha+\sin^2\alpha)}{2l}$$

$$= \frac{2-e^2}{2l} = \text{Constant}$$

(b) PSP' is focal chord of conic



$$\frac{l}{r} = 1 + e\cos\theta \text{ —— ①}$$

eqn of tangent at $P(\alpha)$ is

$$\frac{l}{r} = e\cos\theta + \cos(\theta-\alpha) \text{ —— ②}$$

eqn of tangent at $P'(\pi+\alpha)$ is

$$\frac{l}{r} = e\cos\theta + \cos(\theta-(\pi+\alpha))$$

$$\frac{l}{r} = e\cos\theta - \cos(\theta-\alpha) \text{ —— ③}$$

Intersection of ② and ③ is given by ② + ③

$$\Rightarrow \frac{2l}{r} = 2e\cos\theta$$

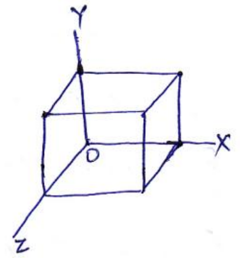
$\frac{l}{r} = e\cos\theta$ which is eqn of directrix of conic $\frac{l}{r} = 1+e\cos\theta$.

⇒ Showing that tangent at P and P' intersect on directrix.

4. (a) Coaxial System of Spheres : If any two members of a system of spheres have same radical plane, such system is called Coaxial system of spheres.

Equation of Coaxial system in simplest form is

$$x^2+y^2+z^2+2\lambda x+d=0.$$



(b) $O(0,0,0)$ is the centre of cube.

Planes parallel to faces of cube are X, Y, Z planes.

If each edge of cube is $2a$ then equation of 6 faces will be $x=a$, $x=-a$, $y=a$, $y=-a$, $z=a$, $z=-a$.

Let $P(f,g,h)$ be variable point.

Distance of $P(f,g,h)$ from face $x=a$ is $\frac{f-a}{\sqrt{1}} = f-a$

" " " " " $x=-a$ is $f+a$

" " " " " $y=a$ is $g-a$

" " " " " $y=-a$ is $g+a$

" " " " " $z=a$ is $h-a$

" " " " " $z=-a$ is $h+a$

Sum of the squares of distances of point P from all 6 faces

$$\Rightarrow (f-a)^2+(f+a)^2+(g-a)^2+(g+a)^2+(h-a)^2+(h+a)^2 = \text{Constant (Given)}$$

$$f^2+a^2-\cancel{2af}+f^2+a^2+\cancel{2af}+g^2+a^2-\cancel{2ga}+g^2+a^2+\cancel{2ga}+h^2+a^2-\cancel{2ah}+h^2+a^2+\cancel{2ah}=K^2$$

$$2f^2+2g^2+2h^2+6a^2=K^2$$

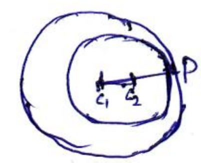
$$f^2+g^2+h^2=\left(\frac{K^2}{2}-3a^2\right)$$

Locus of Point $P(f,g,h)$ is given by

$$x^2+y^2+z^2=\left(\frac{K^2}{2}-3a^2\right)$$ which is a Sphere.

5. Radius and Centre of sphere $x^2+y^2+z^2=64$ is

$PC_1=8$, $C_1=(0,0,0)$



Radius and centre of sphere $x^2+y^2+z^2-12x+4y-6z+48=0$

is $PC_2=\sqrt{36+4+9-48}=1$ $C_2=(6,-2,3)$

Distance between Centres C_1 and C_2 is $C_1C_2=\sqrt{(6-0)^2+(-2-0)^2+(3-0)^2}$

$=\sqrt{36+4+9}=7$

and Difference of Radii $= PC_1-PC_2=8-1=7$

$\because$ $PC_1-PC_2=C_1C_2$ ie Difference between Centres of Spheres = Difference of Radii of spheres

Hence Two spheres touch internally.

Let $P(\alpha,\beta,\gamma)$ is point of contact.

eq^n of tangent plane at (α,β,γ) of two spheres will be

Now planes ① and ② will be same so compairing the Coff. of x, y, z and Constant term in eqⁿ ① and ② we have

$$\frac{\alpha-6}{\alpha} = \frac{\beta+2}{\beta} = \frac{\gamma-3}{\gamma} = \frac{-6\alpha+2\beta-3\gamma+48}{-64} = k$$

Now $\frac{\alpha-6}{\alpha} = k \Rightarrow 1-\frac{6}{\alpha} = k \Rightarrow \alpha = \frac{6}{1-k}$

$\frac{\beta+2}{\beta} = k \Rightarrow 1+\frac{2}{\beta} = k \Rightarrow \beta = \frac{-2}{1-k}$

$\frac{\gamma-3}{\gamma} = k \Rightarrow 1-\frac{3}{\gamma} = k \Rightarrow \gamma = \frac{3}{1-k}$

$\frac{-6\alpha+2\beta-3\gamma+48}{-64} = k \Rightarrow -6\alpha+2\beta-3\gamma+48 = -64k$

Put α, β, γ from above

$$\Rightarrow -6\left(\frac{6}{1-k}\right)+2\left(\frac{-2}{1-k}\right)-3\left(\frac{3}{1-k}\right)+48 = -64k$$

$$\Rightarrow -36-4-9+48(1-k)+64k(1-k) = 0$$

$$\Rightarrow -49+48-48k+64k-64k^2 = 0$$

$$\Rightarrow 64k^2-16k+1 = 0$$

$$\Rightarrow (8k-1)^2 = 0 \Rightarrow k = 1/8$$

$\alpha = \frac{6}{1-k} = \frac{6}{1-1/8} = \frac{48}{7}$, $\beta = \frac{-2}{1-k} = \frac{-2}{1-1/8} = \frac{-16}{7}$, $\gamma = \frac{3}{1-k} = \frac{3}{1-1/8} = \frac{24}{7}$

Point of Contact will be $\left(\frac{48}{7}, \frac{-16}{7}, \frac{24}{7}\right)$.

6. (a) Equation of any line through (α, β, γ) is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r \quad \text{——— ①}$$

Any point on this line will be $(\alpha+lr, \beta+mr, \gamma+nr)$

Intersection of line ① with $z=0$ is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{-\gamma}{n} \Rightarrow x = \alpha-\frac{l\gamma}{n}, \quad y = \beta-\frac{m\gamma}{n}$$

so intersection point will be $\left(\alpha-\frac{l\gamma}{n}, \beta-\frac{m\gamma}{n}, 0\right)$

This point lie on $y^2 = 4ax$ if

$$\left(\beta-\frac{m\gamma}{n}\right)^2 = 4a\left(\alpha-\frac{l\gamma}{n}\right)$$

$$(\beta n-m\gamma)^2 = 4an(n\alpha-l\gamma) \quad \text{——— ②}$$

Eqⁿ of Cone can be obtained by eliminating l, m, n from ① and ② which will be locus of line ①.

from ① $l = \frac{x-\alpha}{r}$, $m = \frac{y-\beta}{r}$, $n = \frac{z-\gamma}{r}$ put in ②

$$\left(\beta\cdot\frac{z-\gamma}{r}-\gamma\cdot\frac{y-\beta}{r}\right)^2 = 4a\cdot\frac{z-\gamma}{r}\left(\alpha\cdot\frac{z-\gamma}{r}-\gamma\cdot\frac{x-\alpha}{r}\right)$$

$$\{\beta(z-\gamma)-\gamma(y-\beta)\}^2 = 4a(z-\gamma)\{\alpha(z-\gamma)-\gamma(x-\alpha)\}$$

$$(\beta z-\cancel{\beta\gamma}-y\gamma+\cancel{\beta\gamma})^2 = 4a(z-\gamma)(\alpha z-\cancel{\alpha\gamma}-x\gamma+\cancel{\alpha\gamma})$$

(b) We know that the condition for the plane $ux + vy + wz = 0$ cuts the cone $f(x,y,z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ in perpendicular lines if

$$(a+b+c)(u^2+v^2+w^2) = f(u,v,w) \quad \text{——} \ (1)$$

Here plane is $ax + by + cz = 0$ $\Rightarrow (u,v,w) = (a,b,c)$

and Cone is $yz + zx + xy = 0$ $\Rightarrow f(x,y,z) = yz + zx + xy$

Hence $a = b = c = 0$

put in condition (1) we get

$$0 \cdot (u^2+v^2+w^2) = f(a,b,c)$$

$$\Rightarrow f(a,b,c) = 0$$

$$\Rightarrow bc + ac + ab = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$ which is the required Condition.

<u>Second Method</u>

Plane $ax + by + cz = 0$ —— (1)

Cone $yz + zx + xy = 0$ —— (2)

Plane (1), cuts the cone (2) in lines. Let one line is $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ —— (3)

Line (3) lie on plane (1) and cone (2) Hence

$$al + bm + cn = 0 \quad \text{——} \ (3)$$

$$mn + ln + lm = 0 \quad \text{——} \ (4)$$

from (3) $n = -\left(\frac{al+bm}{c}\right)$ put in (4)

$$-m\left(\frac{al+bm}{c}\right) - l\left(\frac{al+bm}{c}\right) + lm = 0$$

$$\Rightarrow -alm - bm^2 - al^2 - blm + clm = 0$$

$$\Rightarrow -a\frac{l}{m} - b - a\frac{l^2}{m^2} - b\frac{l}{m} + c\frac{l}{m} = 0$$

$$\Rightarrow (-a)\frac{l^2}{m^2} + (c-b-a)\frac{l}{m} - b = 0$$

This is quadratic equation in $\frac{l}{m}$ gives two values say $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$

then $\frac{l_1}{m_1} \cdot \frac{l_2}{m_2} = \frac{b}{a}$ $\Rightarrow \frac{l_1 l_2}{1/a} = \frac{m_1 m_2}{1/b} = \frac{n_1 n_2}{1/c}$ (Similarly) —— (5)

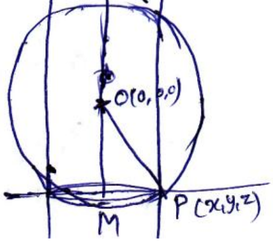
Now two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) are perpendicular if $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \quad (\text{from } (5))$$

7. (a) Radius of Right Circular cylinder

= Radius of Guiding curve

= Radius of Circle $x^2+y^2+z^2=9$, $x-y+z=3$



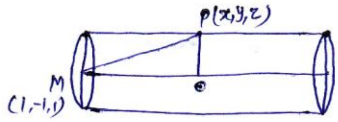
Radius of sphere $x^2+y^2+z^2=9$ is 3 ie $OP = 3$

Length of perpendicular from $O(0,0,0)$ to plane $x-y+z=3$ is $\left|\frac{0-3}{\sqrt{1+1+1}}\right| = \sqrt{3} = OM$

Radius of circle is $PM = \sqrt{OP^2 - OM^2} = \sqrt{9-3} = \sqrt{6}$

eq^n of Line OM is $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1} = r$ (Axis of Cylinder)

Any pt. of this line $(r, -r, r)$



this pt. lie on plane $x-y+z=3$

$\Rightarrow \quad r + r + r = 3 \quad \Rightarrow r = 1$

$M = (1,-1,1)$

$PM = \sqrt{(x-1)^2 + (y+1)^2 + (z-1)^2}$

MO = Projection of PM on Axis OM

$= \frac{(x-1)}{\sqrt{3}} - \frac{(y+1)}{\sqrt{3}} + \frac{(z-1)}{\sqrt{3}}$

$OP =$ Radius $= \sqrt{6}$

$PM^2 = OM^2 + OP^2$

$(x-1)^2 + (y+1)^2 + (z-1)^2 = \left(\frac{x-1}{\sqrt{3}} - \frac{y+1}{\sqrt{3}} + \frac{z-1}{\sqrt{3}}\right)^2 + 6$

$x^2+y^2+z^2-2x+2y-2z+3 = \frac{1}{3}(x-y+z-3)^2 + 6$

$3x^2+3y^2+3z^2 - \cancel{6x} + \cancel{6y} - \cancel{6z} + \cancel{9} = x^2+y^2-2xy+z^2+\cancel{9} - \cancel{6z} + 2xz - \cancel{6x} - 2yz + \cancel{6y} + 18$

$2x^2+2y^2+2z^2+2xy-2zx+2yz-18=0$

$x^2+y^2+z^2+xy+yz-zx-9=0$

(b) Z-Axis $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$

Generators are parallel to Z-Axis

equation of generator through (α, β, γ) is

$\frac{x-\alpha}{0} = \frac{y-\beta}{0} = \frac{z-\gamma}{1} = r$

Any point on this generator is $(\alpha, \beta, \gamma + r)$

This point lie on curve $ax^2+by^2 = 2z$, $lx+my+nz=p$

if $a\alpha^2 + b\beta^2 = 2(\gamma + r)$ —— (A)

$l\alpha + m\beta + n(\gamma + r) = p \quad \Rightarrow n(\gamma+r) = p - l\alpha - m\beta \Rightarrow \gamma + r = \frac{p - l\alpha - m\beta}{n}$

Put $(\gamma + r)$ in eq^n (A)

$a\alpha^2 + b\beta^2 = \frac{2}{n}(p - l\alpha - m\beta)$

$n(a\alpha^2 + b\beta^2) = 2p - 2l\alpha - 2m\beta = 0$

$n(a\alpha^2 + b\beta^2) + 2l\alpha + 2m\beta - 2p = 0$

Locus of (α, β, γ) is

$n(ax^2+by^2) + 2lx + 2my - 2p = 0$

(8) Eqⁿ of Tangent plane to Conicoid $ax^2+by^2+cz^2=1$

is $lx+my+nz = \sqrt{l^2/a+m^2/b+n^2/c}$ —— ①

Intersection of plane ① with X Axis

X Axis is $y=0, z=0$, Intersection with ① is given by

$$lx + m\cdot 0 + n\cdot 0 = \sqrt{l^2/a+m^2/b+n^2/c}$$

$$x = \frac{\sqrt{l^2/a+m^2/b+n^2/c}}{l}$$

so Point $P = \left(\frac{\sqrt{l^2/a+m^2/b+n^2/c}}{l}, 0, 0\right)$

Similarly Intersection of ① with Y Axis, Z Axis is

Point $Q = \left(0, \frac{\sqrt{l^2/a+m^2/b+n^2/c}}{m}, 0\right)$

$R = \left(0, 0, \frac{\sqrt{l^2/a+m^2/b+n^2/c}}{n}\right)$

Now Centroid of ΔPQR is $\left(\frac{1}{3l}\sqrt{l^2/a+m^2/b+n^2/c}, \frac{1}{3m}\sqrt{l^2/a+m^2/b+n^2/c}, \frac{1}{3n}\sqrt{l^2/a+m^2/b+n^2/c}\right)$

Locus of Centroid of ΔPQR will be given by

$\frac{1}{3l}\sqrt{l^2/a+m^2/b+n^2/c} = x \Rightarrow \frac{1}{9l^2}\left(\frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c}\right) = x^2 \Rightarrow 9l^2x^2 = \frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c} \Rightarrow \frac{9l^2}{a} = \frac{1}{ax^2}\left(\frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c}\right)$

$\frac{1}{3m}\sqrt{l^2/a+m^2/b+n^2/c} = y \Rightarrow \frac{1}{9m^2}\left(\frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c}\right) = y^2 \Rightarrow 9m^2y^2 = \frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c} \Rightarrow \frac{9m^2}{b} = \frac{1}{by^2}\left(\text{"} \right)$

$\frac{1}{3n}\sqrt{l^2/a+m^2/b+n^2/c} = z \Rightarrow \frac{1}{9n^2}\left(\frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c}\right) = z^2 \Rightarrow 9n^2z^2 = \frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c} \Rightarrow \frac{9n^2}{c} = \frac{1}{cz^2}\left(\text{"} \right)$

Adding 3 Above we get

$$9\left(\frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c}\right) = \left(\frac{l^2}{a}+\frac{m^2}{b}+\frac{n^2}{c}\right)\left(\frac{1}{ax^2}+\frac{1}{by^2}+\frac{1}{cz^2}\right)$$

$$\boxed{\frac{1}{ax^2}+\frac{1}{by^2}+\frac{1}{cz^2} = 9}$$

Received

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